RAN-2103000206020034

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T.Y.B.Sc. (Sem. VI) Examination October - 2023

Mathematics : Paper - MTH-604 : Real Analysis-IV

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B.Sc. (Sem. VI)**

Name of the Subject :
☛ **Mathematics : Paper - MTH-604 : Real Analysis-IV**

Subject Code No.: **2103000206020034**

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

Q. 1 Answer the following as directed : (Any FIVE) (10)

(1) Prove that $E \subseteq \overline{E}$; for the subset E of a metric space $\langle M, \rho \rangle$.

(2) Prove that $\left\{\frac{22}{23}, \frac{23}{22}\right\}$ is a closed subset of the metric space $\mathbb{R}^1$.

(3) Justify : If A and C are connected subsets of a metric space M. and $A \subset B \subset C$, then B is also connected.

(4) Define the Diameter of a Bounded Set in a Metric Space. Find the diameter of a set in [20,23] in the metric space $\mathbb{R}_d$.

(5) Justify : $[2023, \infty)$ is not a complete subset of the metric space $\mathbb{R}^1$.

(6) State Picard's Fixed - Point Theorem.

(7) Define Compact Metric Space. Give an example of a compact subset of $\mathbb{R}_d$.

(8) State the Finite - Intersection Property.

Q. 2 **Answer any Two of the following :** **(10)**

(a) Define Limit Point of a subset of a Metric Space. If the point x in M is a limit point of E, then prove that every open ball $B[x; r]$; for any $r > 0$; about x contains at least one point of E.

(b) Prove that an arbitrary intersection of closed sets in any metric space is closed. What do you say about an infinite union of closed subsets in a metric space?

(c) Let f be a continuous real - valued function on the metric space M. Let A be the set of all $x \in M$ such that $f(x) \geq 0$. Then prove that A is closed.

Q. 3 **Answer any Two of the following:** **(10)**

(a) If A_1 and A_2 are connected subsets of a metric space $\langle M, \rho \rangle$ and $A_1 \cap A_2 \neq \phi$, then prove that $A_1 \cup A_2$ is also connected.

(b) Define Totally Bounded Set in Metric Space. Prove that a totally bounded set in the metric space $\mathbb{R}_d$ is finite.

(c) (i) Give an example of connected set in the metric spaces $\mathbb{R}^1$ and $\mathbb{R}_d$.

(ii) Prove that $\{2022, 2023\}$ is a totally bounded subset of the metric space $\mathbb{R}^1$.

Q. 4 **Answer any Two of the following :** **(10)**

(a) Define Complete Metric Space. Prove that $\mathbb{R}_d$ is the complete metric space.

(b) Prove that a closed subset of a complete metric space is complete.

(c) If a mapping $T : \langle [0, 1], |\cdot| \rangle \to \langle [0, 1], |\cdot| \rangle$ such that $|T'(x)| \leq \alpha$; for some $\alpha \in [0, 1)$ and for every $x \in [0, 1]$; then prove that T is contraction.

Q. 5 **Answer any Two of the following :** **(10)**

(a) Is the metric space $\mathbb{R}_d$ compact? Justify your answer. If the metric space M is compact, then prove that every sequence of points M has a subsequence converging to a point in M.

(b) Let A be a closed subset of a compact metric space $\langle M, \rho \rangle$. Then prove that $\langle A, \rho \rangle$ is compact.

(c) State the Heine - Borel Property. Let M be a compact metric space and $\mathfrak{I}$ be a family of closed subsets of M having the Finite - Intersection property. Prove that $\bigcap_{F \in \mathfrak{I}} F \neq \phi$.
